7.2

A Rational Shift in Behavior

Translating Rational Functions

LEARNING GOALS

In this lesson, you will:

- Analyze rational functions with a constant added to the denominator.
- Compare rational functions in different forms.
- Identify vertical asymptotes of rational functions.

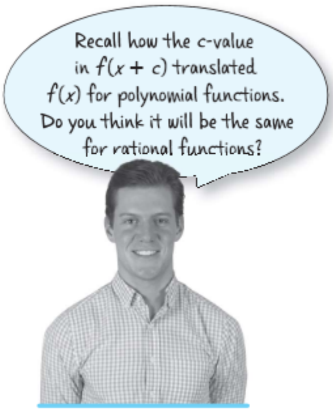
When cars were first built, they all had manual transmissions. This means the drivers had to press on a clutch and shift gears when starting the car, accelerating, or going up an incline. In the 1940's automatic transmission cars were introduced. The transmission was designed so that the gears shift automatically for the driver as the car accelerates or decelerates. Today, automatic transmission cars make up more than 90% of the cars on the road. Ten percent is a pretty small number, but that still amounts to millions of drivers who choose to manually shift gears while driving. Why would they choose a manual transmission?

Manual transmissions are in less demand, so the lower price tag often attracts drivers. They generally get better gas mileage than automatics, adding to the savings over time. Repairs are usually cheaper, too, as the transmission is less complicated. Some drivers also prefer the control that the manual transmission cars offer, especially having the ability to choose the gear when in poor weather or road conditions.

Have you driven a car? Was it an automatic or manual transmission?

PROBLEM 1 Shifty Behavior, Take 1

Recall from *A Rational Existence* that the reciprocal of power functions have a vertical asymptote at $x = 0$ and a horizontal asymptote at $y = 0$. The domain is all real numbers except for 0, because division by 0 is undefined.



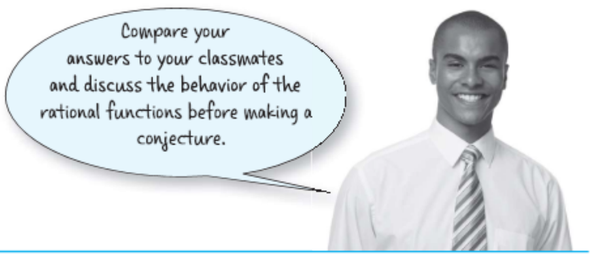
In this problem you will use a graphing calculator to explore rational functions of the form

$g(x) = \frac{1}{x - c}$ for a constant value c.

1. Consider the table shown.

   a. Identify the vertical asymptote, horizontal asymptote, domain, and range for the given c-values. Then choose different positive and negative c-values to complete the table.

c-value	$g(x) = \frac{1}{x - c}$	Vertical Asymptote(s)	Horizontal Asymptote(s)	Domain	Range
1	$g(x) = \frac{1}{x - 1}$				
−2	$g(x) = \frac{1}{x + 2}$				



b. Determine the general formula to identify the vertical asymptote of a rational function in the form $g(x) = \frac{1}{x - c}$. Explain your reasoning.

c. What generalization(s) can you make about the c-value and the domain? The range?

d. What effect does changing the c-value have on the function's end behavior? Explain your reasoning.

2. Without using a graphing calculator, determine the domain, range, and vertical and horizontal asymptotes of each rational function.

a. $f(x) = \frac{10}{x}$

Domain:

Range:

Vertical Asymptote:

Horizontal Asymptote:

b. $g(x) = \frac{1}{x + 10}$

Domain:

Range:

Vertical Asymptote:

Horizontal Asymptote:

c. $j(x) = 10x$

Domain:

Range:

Vertical Asymptote:

Horizontal Asymptote:

d. $g(x) = \frac{1}{x - 10}$

Domain:

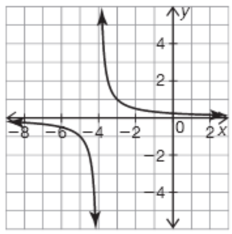
Range:

Vertical Asymptote:

Horizontal Asymptote:

3. Write the rational function(s) from the graph, table, or description provided. Explain your reasoning.

a.



Function: ________________

Explanation:

b. Vertical asymptote at $x = 3$ and a horizontal asymptote at $y = 0$.

Function 1: ________________

Function 2: ________________

Explanation:

c. Domain: All Real Numbers except $x = 7$
Range: All Real Numbers except $y = 0$

Function 1: ________________

Function 2: ________________

Explanation:

d.

x	y
−2	−0.5
−1	$-\frac{2}{3}$
0	−1.0
1	−2.0
2	undefined
3	2.0
4	1.0

Function: ______________

Explanation:

e.

x	y
−2	−3.0
−1	undefined
0	3.0
1	1.5
2	1
3	0.75
4	0.60

Function: ______________

Explanation:

4. Compare the effect that changing the *c*-value has on the reciprocal function to the effect that the *C*-value has when preformed on any polynomial function $f(x)$ to form $g(x) = f(x - C)$.

PROBLEM 2 Shifty Behavior, Take 2

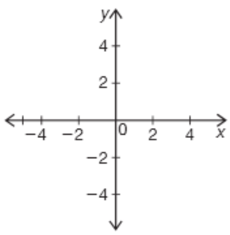
Recall that rational functions are any functions of the form $f(x) = \frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomial functions, and $Q(x) \neq 0$. So far you have only studied a small subset of all rational functions. Let's consider the structure of the rational functions that you have explored so far.

1. Why have the horizontal asymptotes occurred at $y = 0$? Do you think all rational functions will have a horizontal asymptote at $y = 0$? Explain your reasoning.

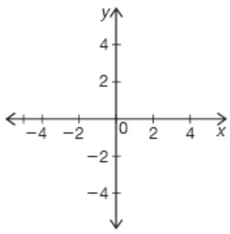
2. What determines a vertical asymptote? Do you think that a rational function could have more than one vertical asymptote?

3. Without using a graphing calculator, sketch each function.

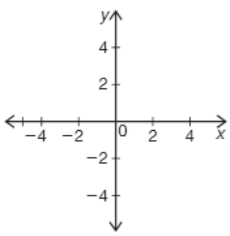
a. $y = \frac{1}{x}$



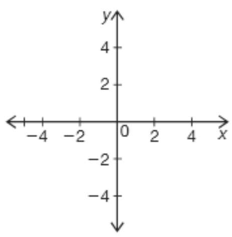
b. $y = \frac{1}{x - 2}$



c. $y = \frac{1}{(x - 2)^2}$

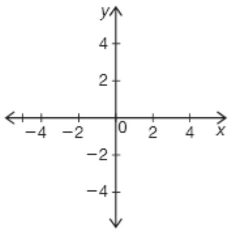


d. $y = \frac{-1}{(x - 2)^2}$

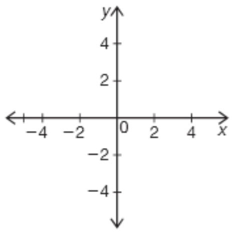


4. Without using a graphing calculator, sketch each function.

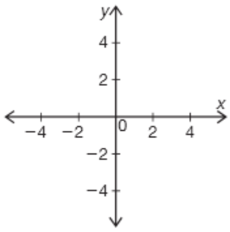
a. $y = \frac{1}{x}$



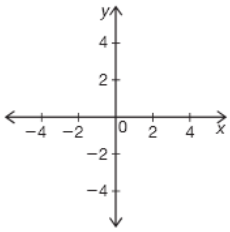
b. $y = \frac{1}{x^3}$



c. $y = \frac{1}{(x + 3)^3}$



d. $y = \frac{-1}{(x + 3)^3}$



5. How would the graphs in Question 3 change for a greater even power? Explain your reasoning.

6. How would the graphs change in Question 4 for a larger odd power? Explain your reasoning.

7

PROBLEM 3 Shifty Behavior, Take 3

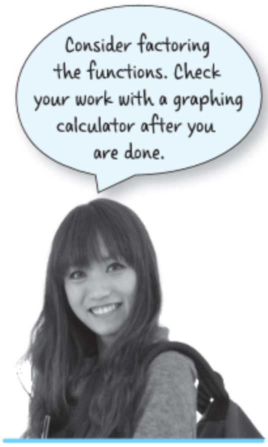
In the previous problems in this chapter, you analyzed rational functions with just 1 vertical asymptote. The vertical asymptote occurred at the value for which the denominator was zero.

1. Without graphing, determine the number of vertical asymptotes for each function. Show all work and explain your reasoning.

   a. $f(x) = \frac{4}{x^2 + 4}$

   b. $g(x) = \frac{4}{x^2 - 4}$

   c. $h(x) = \frac{4}{x^2 + 4x + 4}$



Recall from the Fundamental Theorem of Algebra that a function of degree n has n zeros. Some of the zeros may be imaginary. Therefore, it follows that the reciprocal of a function of degree n can have at most n vertical asymptotes.

2. Sarah determines the vertical asymptotes for the function $f(x) = \frac{1}{2x^2 - 14x - 16}$.

> Sarah
>
> The terms in the denominator have a common factor of 2, so I factored it out first. Then I factored the remaining quadratic.
>
> $f(x) = \frac{1}{2(x^2 - 7x - 8)} = \frac{1}{2(x - 8)(x + 1)}$
>
> Vertical asymptotes occur when the denominator is zero. So, the asymptotes will occur when $x - 8 = 0$ and when $x + 1 = 0$.
>
> Therefore, the asymptotes occur at $x = 8$ and $x = -1$.

Is Sarah correct? Explain your reasoning.

3. Analyze each rational function. Use algebra to determine the vertical asymptotes.

a. $f(x) = \frac{5}{7x - 35}$

b. $g(x) = \frac{1}{x(x - 2)(2x + 3)}$

c. $h(x) = \frac{10}{x^2 - 3x - 10}$

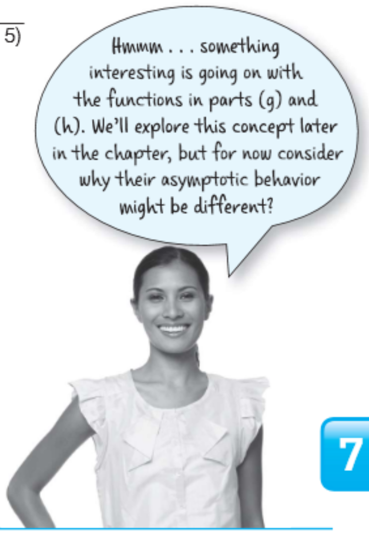
d. $h(x) = \frac{x}{2x^2 + 9x + 4}$

e. $h(x) = \frac{7}{x^4 - 1}$

f. $f(x) = \frac{2}{x^2 + 2}$

g. $h(x) = \frac{x - 2}{x - 2}$

h. $g(x) = \frac{x + 2}{(x + 2)(x - 5)}$



i. Use a graphing calculator to check your answers to Questions 3 by graphing and then by analyzing the table of values.

4. Determine 2 different rational functions with the characteristics given.

a. vertical asymptotes at $x = 3$, $x = -1$, and $x = 0$

b. vertical asymptotes at $x = \frac{1}{2}$ and $x = 2$

Be prepared to share your solutions and methods.